approximate forms were compared to some available accepted analytical solutions, and to experimental data for a number of different decelerating and accelerating flows with variable surface flux, as a check and a demonstration of expected accuracy of the approximate solutions.

One of the comparisons of predictions to experimental data resulted in the suggestion that an empirical modification be made to equation (11), namely the use of the power 0.12 instead of 0.20 in its denominator.

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Negative heat transfer in separated flows

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1. INTRODUCTION

More often than not, time-dependent flows reveal unexpected instantaneous features couched under the time averaged. A classic case is that of the time-dependent, large-scale coherent structure found in a turbulent mixing layer [1], a flow regarded disorderly and featureless in time-averaged measurements. As another and more recent example, it was pointed out [2] that the vortex street behind a body can separate the total temperatures into hot and cold spots around vortices, although the steady data only disclose the colder wake. The latter is a time-averaged 'footprint' left by the former, the instantaneous total temperature separation; the time-averaged, mean colder wake manifests itself as the negative recovery factor on the rearward portion of the cylinder surface, an effect first discovered by Eckert and Weise [3] for a thermally insulated cylinder in air.

For a heat-conducting cylinder, then, even when the bulk fluid temperature is warmer than the surface temperature, the heat can be transferred locally from the rearward surface to the adjacent fluid. It is, therefore, contrary to the conventional expectation that convective heat transfer takes place in the same direction as the surface to bulk fluid temperature difference. Such '*negative*' heat transfer, obviously a time-averaged effect, conceals once again the more intricate transient. Viewed from the point of the time-dependent flow, transient heat transfer is found to fluctuate between *negative* and *positive* values, leaving an imprint of the negative heat transfer as its mean; the mechanism is subtler than to be superficially expected from the aforementioned instantaneous total temperature separation. This is the subject to be discussed below.

2. DISCUSSION AND COMPUTATIONAL RESULTS

For the present purpose, it is of course convenient to investigate the instantaneous *static* temperature distribution rather than the total temperature treated in ref. [2]. Consider a cylinder held at uniform wall temperature (T_w) immersed in a compressible fluid, the upstream static temperature (T_∞) of which is higher than T_w. (Therefore, the upstream total temperature ($T_{t\infty}$) is also higher than T_w.)

In the forward portion of the cylinder where the flow is attached, the heat always transfers from the warmer fluid to the colder surface. Consider, however, the rearward portion where the boundary layer is separating from one side and a vortex is being formed (Fig. 1). As the vortex rolls up and gains strength, the static pressure at the vortex center continues to fall; the pressure drop is induced to counterbalance

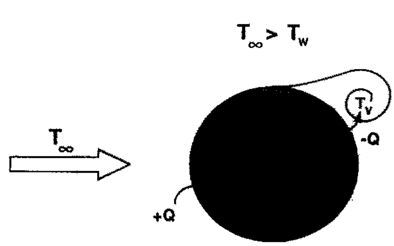

Fig. 1. Sketch of negative heat transfer.

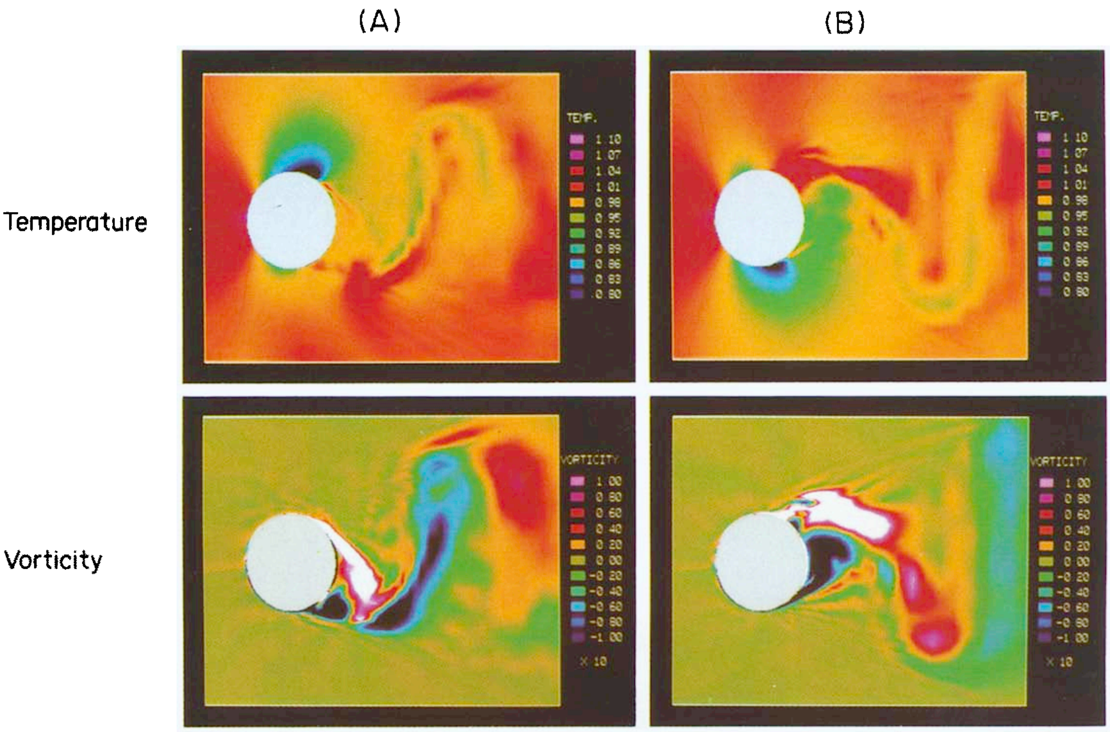

FIG. 3. Instantaneous temperature and vorticity distribution corresponding to (A) and (B) of Fig. 2.

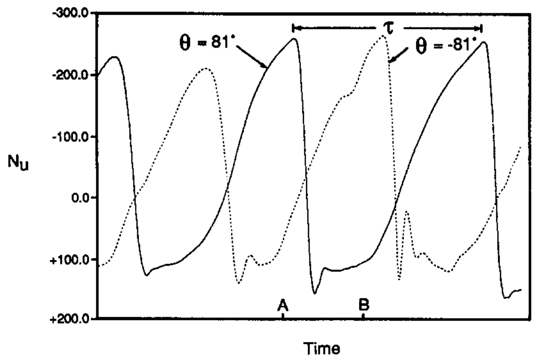

FIG. 2. Time-traces of the local Nusselt number: $T_{t\infty} = 550°R$; $T_\infty = 538°R$; $T_w = 500°R$.

the increase in the centrifugal force, which, in turn is caused by the increase in the swirl velocity around the evolving vortex. Corresponding to the decrease in the static pressure at the vortex center, the static temperature there (T_v) starts to drop almost isentropically and drops to such an extent that T_v might become lower than T_w. Then and there, the heat starts to flow from the body to the fluid. (As will be seen, the presence of the cylinder wall, though it is in proximity to the vortex, does not substantially alter the above description.)

As the vortex is washed away downstream, the flow reattaches to the body and the heat now flows from the fluid to the body. Therefore, for a body shedding a vortex street, the surface heat transfer coefficients in the separated region are expected to fluctuate periodically between positive and negative values, with a frequency equal to the shedding frequency. Furthermore, due to the alternate appearance of vortices from two sides of the body, the time trace of the heat transfer coefficients at positions of $\phi = \theta$ and $-\theta$ should differ from each other by a phase shift of one-half of the period of vortex shedding (τ).

This is illustrated in Fig. 2, a result based on a full numerical simulation discussed in ref. [4]; the abscissa is the time, the ordinate the local Nusselt number defined by

$$Nu = \frac{2a}{\Delta T}\frac{\partial T}{\partial r}\bigg|_{r=a}$$

when a is the radius of the cylinder, $\Delta T = |T_w - T_{t\infty}|$, T the static temperature and r the radial distance. According to this definition, when $Nu < 0$, the heat flows from the body to the fluid, and for $Nu > 0$, vice versa. The fluctuation between positive and negative values of Nu and the phase shift by $\tau/2$ between $\phi = 81°$ and $-81°$ are all evident (the reason why the fluctuation, although nearly periodic, is not perfectly so is due to the inherent characteristics of the time-marching method adopted in the numerical simulation). In Fig. 2, the fluid is air, the upstream Mach number 0.35, the Reynolds number based on the diameter 1.4×10^5, and T_w is maintained to be constant along the cylinder surface.

The snapshots of the static temperature and vorticity distribution at a moment corresponding to (A) of Fig. 2 and (B), approximately half a period after (A), are shown in Fig. 3; both are nondimensionalized, the former by T_∞ and the latter by the far upstream velocity and the cylinder diameter; the flow field including those of the separated region is treated as laminar, the other details related to numerical computation and construction of color graphics being discussed in ref. [2]. In Fig. 3(A), the positive or clockwise vorticity† is evolving from the upper half portion of the cylinder and this is accompanied by colder static temperatures around $\theta = 90°$; in Fig. 3(B), the region of the negative or counter-clockwise vorticity has spread over the lower half, with colder spots now around $\theta = -90°$. (The separation of *total* temperature around vortices is confirmed to occur in a manner similar to that discussed in ref. [2] for a thermally insulated cylinder.) A comparison between Figs. 2 and 3 confirms the earlier expectation: the alternating appearance and disappearance of the cold spots correspond to the negative and positive values of Nu, respectively.

The negative values of Nu are expected to exist only in the limited range; as T_w is reduced substantially, there comes a point where T_v, dependent primarily on the upstream condition,‡ starts to exceed T_w, leading to positive Nu at all times. The delineation of such a critical condition and its experimental confirmation call for a further study.

3. CONCLUSION

To sum up, we pointed out that in the separated flow region around a cylinder, transient heat transfer fluctuates between negative and positive values, i.e. from the colder surface to the warmer bulk fluid and vice versa.

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† The white color in the vorticity plots corresponds to the vorticity the intensity of which is out of scale, higher than that of pink.

‡ T_v, properly nondimensionalized, is also dependent upon the cylinder diameter d, because the circulation of the vortex scales with d, and the upstream velocity.

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